Uncertainty Analysis for Multiphase Flow: A Case Study for Horizontal Air-Water Flow Experiments

Felipe Jaloretto da Silva / Marcelo Souza de Castro
Introduction

• To improve the quality of experimental result

• To describe a simple methodology for calculating uncertainties in multiphase experiments

• To identify and to quantify sources of uncertainty for the velocity and pressure results
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Experimental Facilities and Procedures

<table>
<thead>
<tr>
<th>D (mm)</th>
<th>( j_1 ) (m/s) (water)</th>
<th>( j_2 ) (m/s) (air)</th>
<th>Flow Pattern</th>
<th>Nr. of Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.75</td>
<td>0.04 to 6.37</td>
<td>0.30 to 15.87</td>
<td>SS, WS, EB, S, B, A</td>
<td>120</td>
</tr>
<tr>
<td>53.55</td>
<td>0.05 to 2.12</td>
<td>0.25 to 4.32</td>
<td>WS, EB, S</td>
<td>70</td>
</tr>
<tr>
<td>82.25</td>
<td>0.02 to 0.76</td>
<td>0.11 to 1.94</td>
<td>SS, WS, EB, S</td>
<td>129</td>
</tr>
<tr>
<td>106.47</td>
<td>0.10 to 3.72</td>
<td>0.10 to 1.02</td>
<td>SS, WS, EB, S, B</td>
<td>78</td>
</tr>
</tbody>
</table>
Experimental Facilities and Procedures

- Understand the diameter influence mainly on two-phase pressure drop:

<table>
<thead>
<tr>
<th>Slug flow:</th>
<th>Front part</th>
<th>Rear part</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 in</td>
<td><img src="1-in-front.png" alt="Image" /></td>
<td><img src="1-in-rear.png" alt="Image" /></td>
</tr>
<tr>
<td>2 in</td>
<td><img src="2-in-front.png" alt="Image" /></td>
<td><img src="2-in-rear.png" alt="Image" /></td>
</tr>
<tr>
<td>3 in</td>
<td><img src="3-in-front.png" alt="Image" /></td>
<td><img src="3-in-rear.png" alt="Image" /></td>
</tr>
<tr>
<td>4 in</td>
<td><img src="4-in-front.png" alt="Image" /></td>
<td><img src="4-in-rear.png" alt="Image" /></td>
</tr>
</tbody>
</table>

\[ j_1 = 1.03 \text{ m/s} \quad \text{and} \quad j_2 = 2.18 \text{ m/s} \]

\[ j_1 = 0.13 \text{ m/s} \quad \text{and} \quad j_2 = 2.80 \text{ m/s} \]

\[ j_1 = 0.09 \text{ m/s} \quad \text{and} \quad j_2 = 1.49 \text{ m/s} \]

\[ j_1 = 0.57 \text{ m/s} \quad \text{and} \quad j_2 = 0.51 \text{ m/s} \]
Uncertainties

11 steps:
1. Define physical quantity
2. Define a physical model
3. Define a mathematical model
4. Identify the sources of uncertainty
5. Organize the sources and look for cause and effect relations
6. Quantify the variability of each source
7. Reduce to standard uncertainty
8. Determining the combined standard uncertainty
9. Look for correlated input quantities
10. Define reliability
11. Determining expanded uncertainty
Uncertainties

- Water superficial velocity

\[ j_1 = \frac{W_1}{\rho_1 A} \]
Uncertainties

- Air superficial velocity

\[ j_2 = \frac{W_2}{\rho_2 A} = \frac{W_2}{(P_{atm} + P_{est}) A} = \frac{W_2 RT}{(P_{atm} + P_{est}) A} \]
Uncertainties

- Pressure drop (single phase and two-phase)

Calibration

Reproducibility

$\Delta P$
## Uncertainties

### - Quantifying and reduce

<table>
<thead>
<tr>
<th>Uncertainties of meter and sensor calibrations: (type B)</th>
<th>$u(X) = \frac{U(X)}{k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainties of measurements: (type A)</td>
<td>$u(X) = \frac{\sigma(X)}{\sqrt{n}}$</td>
</tr>
<tr>
<td></td>
<td>$u(\bar{X}) = \frac{s(\bar{X})}{\sqrt{n}}$</td>
</tr>
<tr>
<td>Uncertainties of method, instrument resolution or values fluctuation: (type B)</td>
<td>$u(a) = \frac{a}{\sqrt{3}}$ (for rectangular distribution)</td>
</tr>
<tr>
<td></td>
<td>$u(a) = \frac{a}{\sqrt{6}}$ (for triangular distribution)</td>
</tr>
</tbody>
</table>

Where:

- $X$ = physical quantity
- $k$ = coverage factor
- $\sigma$ = standard deviation
- $s$ = biased estimator of standard deviation
- $n$ = number of samples
- $a$ = assigned value, instrument resolution or greater difference
Uncertainties

- Combined standard uncertainty

Area: \( u_c(A) = \sqrt{\left(\frac{\partial A}{\partial D} \cdot u(D)\right)^2} \)

Air density: \( u_c(\rho_2) = \sqrt{\left(\frac{\partial \rho_2}{\partial P_{atm}} \cdot u(P_{atm})\right)^2 + \left(\frac{\partial \rho_2}{\partial P_{est}} \cdot u(P_{est})\right)^2 + \left(\frac{\partial \rho_2}{\partial T} \cdot u(T)\right)^2} \)
Uncertainties

- Combined standard uncertainty

Air superficial velocity:

\[ u_c(j_2) = \sqrt{\left[ \frac{\partial j_2}{\partial W_2} \cdot u(W_2) \right]^2 + \left[ \frac{\partial j_2}{\partial A} \cdot u_c(A) \right]^2 + \left[ \frac{\partial j_2}{\partial \rho_2} \cdot u_c(\rho_2) \right]^2 + u(Rj_2)^2} \]

Water superficial velocity:

\[ u_c(j_1) = \sqrt{\left[ \frac{\partial j_1}{\partial W_1} \cdot u(W_1) \right]^2 + \left[ \frac{\partial j_1}{\partial A} \cdot u_c(A) \right]^2 + \left[ \frac{\partial j_1}{\partial \rho_1} \cdot u(\rho_1) \right]^2 + u(Rj_1)^2} \]

Pressure drop:

\[ u_c(\Delta P) = \sqrt{u(\Delta P)^2 + u(R\Delta P)^2} \]
Uncertainties

- Reliability and degrees of freedom

Expanded uncertainty:

\[ U = k \cdot u_c \]

Coverage factor:

<table>
<thead>
<tr>
<th>( v_{ef} )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{t95} )</td>
<td>13,97</td>
<td>4,53</td>
<td>3,31</td>
<td>2,87</td>
<td>2,65</td>
<td>2,52</td>
<td>2,43</td>
<td>2,37</td>
<td>2,28</td>
<td>2,23</td>
<td>2,20</td>
<td>2,17</td>
</tr>
<tr>
<td>( N_{ef} )</td>
<td>18</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>45</td>
<td>50</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>∞</td>
</tr>
<tr>
<td>( k_{t95} )</td>
<td>2,15</td>
<td>2,13</td>
<td>2,11</td>
<td>2,09</td>
<td>2,07</td>
<td>2,06</td>
<td>2,06</td>
<td>2,05</td>
<td>2,04</td>
<td>2,03</td>
<td>2,025</td>
<td>2,00</td>
</tr>
</tbody>
</table>

Degrees of freedom:

\[ v_{ef} = \frac{u_c^4(y)}{\sum_{i=1}^{N} \frac{u_i^4(y)}{v_i}} \]
Results – Flow Pattern Maps – 1 in
Results – Flow Pattern Maps – 2 in
Results – Flow Pattern Maps – 3 in

- Elongated Bubble
- Stratified Wavy
- Slug
- Stratified
- Transition
Results – Flow Pattern Maps – 4 in

- Elongated Bubble
- Bubble
- Stratified Wavy
- Slug
- Stratified
- Transition

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Results – Two-Phase Pressure Drop

- Proposition for pressure drop variation with diameter

\[- \left( \frac{dP}{dx} \right) = \frac{2}{d} f \rho J^2 = (0.092 \mu_m^{0.2} \rho_m^{0.8} J^{1.8}) d^{-1.2} \]
Results – Two-Phase Pressure Drop

![Graph showing two-phase pressure drop vs. diameter](image)

- $j_1 = 1.0 \text{ m/s}$ and $j_2 = 1.0 \text{ m/s}$
- Equation 31

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Results – Two-Phase Pressure Drop

![Graph showing the relationship between pressure drop and diameter for two-phase flow with inlet velocities j1=2.0 m/s and j2=1.0 m/s. The graph includes data points and error bars, with the legend noting Equation 31.](image-url)
Results – Two-Phase Pressure Drop

![Graph showing two-phase pressure drop](image)

Equation 31

\[ j_1 = 0.2 \text{ m/s} \quad \text{and} \quad j_2 = 0.5 \text{ m/s} \]
Conclusion

1. Uncertainty for the flow pattern maps can help understand transition regions
2. Better knowledge about the pressure drop results
3. Uncertainty validate the homogeneous model as a good approach
4. Reproducibility of the results improves the quantification of the final uncertainties
Acknowledgments
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